B.Tech Degree I & II Semester Examination in Marine Engineering June 2011

MRE 102 ENGINEERING MATHEMATICS II

Time: 3 Hours Maximum Marks: 100

I. (a) Reduce to normal form the following matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$ and hence determine its rank. (7)

(b) Determine for what values of λ and μ, the following equations have (i) no solution; (ii) a unique solution ; (iii) infinite number of solutions.
$x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ (8)

(c) Write down the matrix of the quadratic form

$$x_1^2+2x_2^2--7x_3^2-4x_1x_2+8x_1x_3+5x_2x_3.$$ (5)

OR

II. (a) Prove that the function $f(z)=\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2},\ z\neq 0,\ f(0)=0$ is continuous and that Cauchy-Riemann equations are satisfied at the origin. Yet $f'(z)$ does not exist there. (10)

(b) If $F(\xi)=\int\limits_C \dfrac{4z^2+z+5}{z-\xi}\,dz$ where C is the ellipse $\left(\dfrac{x}{2}\right)^2+\left(\dfrac{y}{3}\right)^2=1$, find the value of (i) $F(3.5)$ (ii) $F(i)$ (iii) $F'(-1)$ (iv) $F''(-i)$. (10)

III. (a) Solve $\dfrac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$. (5)

(b) Find the orthogonal trajectories of the family of confocal conics $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2+\lambda}=1$ where λ is the parameter. (7)

(c) Solve by the method of variation of parameters, $y''-2y'+y=e^x\log x$. (8)

OR

IV. (a) Solve the following:

(i) $\dfrac{d^4x}{dt^4}+4x=0$ (ii) $\dfrac{d^2y}{dx^2}-\dfrac{2dy}{dx}+y=x\,e^x\sin x$. (4 + 6 = 10)

(b) Solve the simultaneous equations $\dfrac{dx}{dt}+5x-2y=t,\ \dfrac{dy}{dt}+2x+y=0$ being given $x=y=0$ when $t=0$. (10)

V. (a) Find the fourier series of $f(x)=x^2,\ -\pi\le x\le\pi$. Hence, find the sum of the series $\dfrac{1}{1^2}-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\dfrac{1}{4^2}+$............ (10)

(b) Express $f(x)=\begin{cases}1 & \text{for } 0\le x\le\pi \\ 0 & \text{for } x>\pi\end{cases}$ as a fourier sine integral and hence evaluate

$$\int_0^\infty \frac{1-\cos\lambda}{\lambda}\sin x\lambda\,d\lambda.$$ (10)

OR

(P.T.O.)

VI. (a) Show that

(i) $\Gamma n = \int_0^1 \left(\log \frac{1}{y}\right)^{n-1} dy (n > 0)$ (5)

(ii) $\beta(p,q) = \int_0^\infty \frac{y^{q-1}}{(1+y)^{p+q}} dy = \int_0^1 \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}} dx$ (10)

(b) Show that $erf(x) = \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \dots\dots\right)$ (5)

VII. Find the Laplace transform of (i) $t^2 \sin at$ (ii) $t \cos at$ (iii) $\frac{\cos at - \cos bt}{t}$

(iv) $e^{4t} \sin 2t \cos t$ (20)

OR

VIII. (a) Find the Inverse Laplace transform of

(i) $\frac{5s+3}{(s-1)+(s^2+2s+5)}$ (ii) $\frac{1}{s(s+a)^3}$ (12)

(b) Solve by the method of transforms, the equation $y''' + 2y'' - y' - 2y = 0$, given $y(0) = y'(0) = 0$ and $y''(0) = 6$. (8)

IX. (a) State Baye's theorem. (2)

(b) Three machines M_1, M_2 and M_3 produce identical items. Of their respective output 5%, 4% and 3% of items are faulty. On a certain day, M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output. (10)

(c) Is the function defined as follows a density function?

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

If so, determine the probability that the variate having this density will fall in the interval (1, 2). (8)

OR

X. (a) A variate X has the probability distribution

x	-3	6	9
$p(X = x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$ and $E(X^2)$ and hence evaluate $E(2X+1)^2$. (6)

(b) If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction. (6)

(c) In a normal distribution, 31% of the items are under 45 and 8% are over 64, find the mean and S.D. of the distribution. (8)

***